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Generalized quantization scheme for two-person non-zero sum games

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Abstract

We proposed a generalized quantization scheme for non-zero sum games which can be reduced to the two existing quantization schemes under an appropriate set of parameters. Some other important situations are identified which are not apparent in the two existing quantization schemes.

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1. Introduction

Game theory stepped into the quantum domain with the success of a hypothetical quantum player over a classical player in a quantum Penny Flip game[1] [3]. Later Eisert *et al* [4] introduced an elegant scheme to deal with non-zero sum games quantum mechanically. In this quantization scheme, the strategy space of the players is a two-parameter set of unitary 2×2 matrices. Starting with a maximally entangled initial state they analysed the well-known Prisoner's Dilemma game and showed that for a suitable quantum strategy the dilemma disappears. They also pointed out a quantum strategy which always wins over all the classical strategies. Later Marinatto and Weber [5] introduced another interesting and simple scheme for the analysis of non-zero sum games in the quantum domain. They gave Hilbert structure to the strategic spaces of the players. Using a maximally entangled initial state they allowed

[1] David Meyer in his paper [1] describes the quantum Penny Flip game through a story of a spaceship which faces a catastrophe. Suddenly a quantum being, Q, appears on the bridge to help save the spaceship. The quantum being Q offers his help to save the spaceship on a condition that, Picard, the captain of the spaceship, beat him in a penny flipping game. According to the game, Picard is to place the penny with head up in a box. Then Q has an option to either flip the penny or leave it unchanged. Then Picard has the same options without having a look at the penny. Finally Q takes the turn with the same options without looking at the penny. If in the end the penny is head up then Q wins; otherwise Picard wins. Captain Picard is an expert of game theory and he knows that this game has no deterministic solution (see [2]). In other words, there exist no such pair of pure strategies from which unilateral change by the player enhances his/her payoff. To Picard's surprise, Q always wins. This happens as Q is capable of playing quantum strategies which is the superposition of head and tail in the two-dimensional Hilbert space.

the players to play their tactics by applying the probabilistic choice of unitary operators. They applied their scheme to an interesting game of Battle of the Sexes and found out the strategy for which both the players can achieve equal payoffs.

Both Eisert and Marinatto's and Weber's schemes gave interesting results for various quantum versions of the games [6–11]. It seems natural to look for a relationship between these two apparently different quantization schemes. In these papers, we have developed a generalized quantization scheme for non-zero sum games. The game of Battle of the Sexes has been used as an example to introduce this quantization scheme which is also applicable to other games. A separate set of parameters is identified for which our scheme reduces to those of Marinatto and Weber [5] and Eisert *et al* [4]. Furthermore, we have identified other interesting situations which are not apparent within the two existing quantization schemes. After a brief introduction to Battle of the Sexes, we have extended Marinatto and Weber's mathematical framework by redefining unitary operators for our generalized quantization scheme.

2. Generalized quantization scheme

Battle of the Sexes is an interesting static game of complete information. In its usual exposition two players, Alice and Bob, try to decide somewhere to spend Saturday evening. Alice wants to go for opera while Bob is interested in watching TV at home and both would prefer to spend the evening together. The game is represented by the following payoff matrix:

$$\text{Alice}\begin{matrix} \\ O \\ T \end{matrix}\overset{\text{Bob}}{\overset{\begin{matrix} O & \quad T \end{matrix}}{\begin{bmatrix} (\alpha, \beta) & (\sigma, \sigma) \\ (\sigma, \sigma) & (\beta, \alpha) \end{bmatrix}}},$$

where O and T represent opera and TV, respectively, and α, β, σ are the payoffs for players for different choices of strategies, such that, $\alpha > \beta > \sigma$. There exist two Nash equilibria, (O, O) and (T, T), in the classical form of the game. In the absence of any communication between Alice and Bob, there is a dilemma as the Nash equilibria (O, O) suit Alice whereas Bob prefers the Nash equilibria (T, T). As a result both players could end up with worst payoff in the case where they play mismatched strategies. Marinatto and Weber [5] presented the quantum version of the game to resolve this dilemma. In our earlier paper, we have further extended their work to remove the worst case payoff situation in Battle of the Sexes [11]. On the other hand Eisert *et al* [4] presented a different scheme to remove dilemma in the game of Prisoner's Dilemma through quantization of the game.

Here we present a generalized quantization scheme by redefining unitary operators in the Marinatto and Weber scheme. Let Alice and Bob be given the following initial state:

$$|\psi_{\text{in}}\rangle = \cos\frac{\gamma}{2}|OO\rangle + \mathrm{i}\sin\frac{\gamma}{2}|TT\rangle. \tag{1}$$

Here $|O\rangle$ and $|T\rangle$ represent the vectors in the strategy space corresponding to opera and TV, respectively and $\gamma \in \left[0, \frac{\pi}{2}\right]$. The strategies of the two players are represented by the unitary operator, U_i, which is of the form

$$U_i = \cos\frac{\theta_i}{2}R_i + \sin\frac{\theta_i}{2}C_i, \tag{2}$$

where $i = 1$ or 2 and R, C are the unitary operators defined as

$$\begin{aligned} R|O\rangle &= \mathrm{e}^{\mathrm{i}\phi_i}|O\rangle, \qquad & R|T\rangle &= \mathrm{e}^{-\mathrm{i}\phi_i}|T\rangle, \\ C|O\rangle &= -|T\rangle, \qquad & C|T\rangle &= |O\rangle. \end{aligned} \tag{3}$$

Following [4], we restrict our treatment to a two-parameter set of strategies for mathematical simplicity. After the application of the strategies, the initial state (1) transforms into

$$|\psi_f\rangle = (U_1 \otimes U_2)|\psi_{\text{in}}\rangle \tag{4}$$

and using equations (2) and (3) the above expression becomes

$$\begin{aligned}|\psi_f\rangle = \cos\frac{\gamma}{2}\bigg[&\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\,\mathrm{e}^{\mathrm{i}(\phi_1+\phi_2)}|OO\rangle - \cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\,\mathrm{e}^{\mathrm{i}\phi_1}|OT\rangle \\ &- \cos\frac{\theta_2}{2}\sin\frac{\theta_1}{2}\,\mathrm{e}^{\mathrm{i}\phi_2}|TO\rangle + \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}|TT\rangle\bigg] \\ &+ \mathrm{i}\sin\frac{\gamma}{2}\bigg[\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\,\mathrm{e}^{-\mathrm{i}(\phi_1+\phi_2)}|TT\rangle + \cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\,\mathrm{e}^{-\mathrm{i}\phi_1}|TO\rangle \\ &+ \cos\frac{\theta_2}{2}\sin\frac{\theta_1}{2}\,\mathrm{e}^{-\mathrm{i}\phi_2}|OT\rangle + \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}|OO\rangle\bigg].\end{aligned} \tag{5}$$

The payoff operators for Alice and Bob are

$$P_A = \alpha P_{OO} + \beta P_{TT} + \sigma(P_{OT} + P_{TO}) \qquad P_B = \alpha P_{TT} + \beta P_{OO} + \sigma(P_{OT} + P_{TO}) \tag{6}$$

where

$$P_{OO} = |\psi_{OO}\rangle\langle\psi_{OO}|, \qquad |\psi_{OO}\rangle = \cos\frac{\delta}{2}|OO\rangle + \mathrm{i}\sin\frac{\delta}{2}|TT\rangle, \tag{7a}$$

$$P_{TT} = |\psi_{TT}\rangle\langle\psi_{TT}|, \qquad |\psi_{TT}\rangle = \cos\frac{\delta}{2}|TT\rangle + \mathrm{i}\sin\frac{\delta}{2}|OO\rangle, \tag{7b}$$

$$P_{TO} = |\psi_{TO}\rangle\langle\psi_{TO}|, \qquad |\psi_{TO}\rangle = \cos\frac{\delta}{2}|TO\rangle - \mathrm{i}\sin\frac{\delta}{2}|OT\rangle, \tag{7c}$$

$$P_{OT} = |\psi_{OT}\rangle\langle\psi_{OT}|, \qquad |\psi_{OT}\rangle = \cos\frac{\delta}{2}|OT\rangle - \mathrm{i}\sin\frac{\delta}{2}|TO\rangle, \tag{7d}$$

with $\delta \in \left[0, \frac{\pi}{2}\right]$. The above payoff operators reduce to that of Eisert's scheme for δ equal to γ, which represents the entanglement of the initial state. And for $\delta = 0$ above operators transform into that of Marinatto and Weber's scheme. In generalized quantization scheme payoff for the players is calculated as[2]

$$\$_A(\theta_1, \phi_1, \theta_2, \phi_2) = \mathrm{Tr}(P_A\rho_f), \qquad \$_B(\theta_1, \phi_1, \theta_2, \phi_2) = \mathrm{Tr}(P_B\rho_f), \tag{8}$$

where $\rho_f = |\psi_f\rangle\langle\psi_f|$ is the density matrix for the quantum state given by (5) and Tr represents the trace of a matrix. Using equations (5), (6), (8) the payoffs for players are obtained as

$$\begin{aligned}\$_A(\theta_1, \phi_1, \theta_2, \phi_2) = \cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}&\left[\eta\sin^2\frac{\gamma}{2} + \xi\cos^2\frac{\gamma}{2} + \chi\cos 2(\phi_1+\phi_2)\sin\gamma - \sigma\right] \\ &+ \sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}\left(\eta\cos^2\frac{\gamma}{2} + \xi\sin^2\frac{\gamma}{2} - \chi\sin\gamma - \sigma\right) \\ &+ \frac{(\alpha+\beta-2\sigma)\sin\gamma - 2\chi}{4}\sin\theta_1\sin\theta_2\sin(\phi_1+\phi_2) + \sigma\end{aligned} \tag{9a}$$

[2] The schemes of Eisert *et al* and Marinatto and Weber can also be compared using the entanglement operator introduced by Eisert *et al* [14] without calculating the payoffs of the players. Take $|\psi_{\text{in}}\rangle = \hat{J}(\frac{\gamma}{2})\,|OO\rangle$, where $\hat{J}(\frac{\gamma}{2}) = \exp(-\mathrm{i}\frac{\gamma}{2}D\otimes D)$ is the entanglement operator. The strategic moves of Alice and Bob are associated with U_1 and U_2, respectively. Execution of players' moves leave the game in a state $(U_1\otimes U_2)\,\hat{J}(\frac{\gamma}{2})|OO\rangle$. Then Alice and Bob forward their qubits to arbiter for measurement and the final state of the game prior to the detection is $|\psi_f\rangle = \hat{J}^\dagger(\frac{\delta}{2})(U_1\otimes U_2)\hat{J}(\frac{\gamma}{2})|OO\rangle$, where $\hat{J}(\frac{\delta}{2}) = \exp(-\mathrm{i}\frac{\delta}{2}D\otimes D)$ is the disentanglement operator. Putting $\delta = \gamma$ gives the original scheme of Eisert *et al* and letting $\delta = 0$ with restriction of U_1 and U_2 as a linear combination of identity operator I, and the flip operator σ_x, the scheme of Marinatto and Weber is retrieved.

$$\$_B(\theta_1,\phi_1,\theta_2,\phi_2) = \cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\left[\xi\sin^2\frac{\gamma}{2} + \eta\cos^2\frac{\gamma}{2} - \chi\cos 2(\phi_1+\phi_2)\sin\gamma - \sigma\right] + \sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}\left(\xi\cos^2\frac{\gamma}{2} + \eta\sin^2\frac{\gamma}{2} + \chi\sin\gamma - \sigma\right) + \frac{(\alpha+\beta-2\sigma)\sin\gamma + 2\chi}{4}\sin\theta_1\sin\theta_2\sin(\phi_1+\phi_2) + \sigma, \tag{9b}$$

where $\xi = \alpha\cos^2\frac{\delta}{2} + \beta\sin^2\frac{\delta}{2}$, $\eta = \alpha\sin^2\frac{\delta}{2} + \beta\cos^2\frac{\delta}{2}$ and $\chi = \frac{(\alpha-\beta)}{2}\sin\delta$. Classical results can easily be found from equations (9*a*), (9*b*) by simply unentangling the initial quantum state of the game, i.e., letting $\gamma = 0$. Furthermore, all the results of both Marinatto and Weber [5] and Eisert *et al* [4] are embedded in these payoffs. For different combinations of δ and $\phi's$ there exist the following possibilities:

Case (a). When $\delta = 0$ and

(i) $\phi_1 = 0, \phi_2 = 0$, then the payoffs for the players from equations (9*a*), (9*b*) reduce to

$$\$_A(\theta_1,\phi_1,\theta_2,\phi_2) = \cos^2\frac{\theta_1}{2}\left[\cos^2\frac{\theta_2}{2}(\alpha+\beta-2\sigma) - \alpha\sin^2\frac{\gamma}{2} - \beta\cos^2\frac{\gamma}{2} + \sigma\right] + \cos^2\frac{\theta_2}{2}\left(-\alpha\sin^2\frac{\gamma}{2} - \beta\cos^2\frac{\gamma}{2} + \sigma\right) + \alpha\sin^2\frac{\gamma}{2} + \beta\cos^2\frac{\gamma}{2} \tag{10a}$$

$$\$_B(\theta_1,\phi_1,\theta_2,\phi_2) = \cos^2\frac{\theta_2}{2}\left[\cos^2\frac{\theta_1}{2}(\alpha+\beta-2\sigma) - \beta\sin^2\frac{\gamma}{2} - \alpha\cos^2\frac{\gamma}{2} + \sigma\right] + \cos^2\frac{\theta_1}{2}\left(-\beta\sin^2\frac{\gamma}{2} - \alpha\cos^2\frac{\gamma}{2} + \sigma\right) + \beta\sin^2\frac{\gamma}{2} + \alpha\cos^2\frac{\gamma}{2}. \tag{10b}$$

These payoffs are the same as found by Marinatto and Weber [5] when the players are applying the identity operators I_1 and I_2 with probabilities $\cos^2\frac{\theta_1}{2}$ and $\cos^2\frac{\theta_2}{2}$, respectively, for the given initial quantum state of the form (1).

(ii) $\phi_1 + \phi_2 = \frac{\pi}{2}$, then equations (9*a*), (9*b*) reduce to

$$\$_A(\theta_1,\phi_1,\theta_2,\phi_2) = \cos^2\frac{\theta_1}{2}\left[\cos^2\frac{\theta_2}{2}(\alpha+\beta-2\sigma) - \alpha\sin^2\frac{\gamma}{2} - \beta\cos^2\frac{\gamma}{2} + \sigma\right] + \cos^2\frac{\theta_2}{2}\left(-\alpha\sin^2\frac{\gamma}{2} - \beta\cos^2\frac{\gamma}{2} + \sigma\right) + \alpha\sin^2\frac{\gamma}{2} + \beta\cos^2\frac{\gamma}{2} + \frac{(\alpha+\beta-2\sigma)}{4}\sin\gamma\sin\theta_1\sin\theta_2, \tag{11a}$$

$$\$_B(\theta_1,\phi_1,\theta_2,\phi_2) = \cos^2\frac{\theta_2}{2}\left[\cos^2\frac{\theta_1}{2}(\alpha+\beta-2\sigma) - \beta\sin^2\frac{\gamma}{2} - \alpha\cos^2\frac{\gamma}{2} + \sigma\right] + \cos^2\frac{\theta_1}{2}\left(-\beta\sin^2\frac{\gamma}{2} - \alpha\cos^2\frac{\gamma}{2} + \sigma\right) + \beta\sin^2\frac{\gamma}{2} + \alpha\cos^2\frac{\gamma}{2} + \frac{(\alpha+\beta-2\sigma)}{4}\sin\gamma\sin\theta_1\sin\theta_2. \tag{11b}$$

In the context of Marinatto and Weber scheme [5, 15] above payoffs for the two players correspond to a situation when the strategies of the players are linear combination of operators I

and flip operator σ_x of the form $O_i = \sqrt{p_i} I + \sqrt{1-p_i}\sigma_x$ with $p_i = \cos^2\frac{\theta_i}{2}$ and initial entangled state is of the form given in equation (1).

Case (b). When $\delta = \gamma$ and

(i) $\phi_1 \neq 0, \phi_2 \neq 0$ then payoffs given by the equations (9*a*), (9*b*), very interestingly, change to the payoffs as if the game has been quantized using Eisert *et al* [4] scheme for the initial quantum state of the form (1). In this situation the payoffs for both the players are

$$\begin{aligned}\$_A(\theta_1,\phi_1,\theta_2,\phi_2) &= \cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\left[\eta_1\sin^2\frac{\gamma}{2} + \xi_1\cos^2\frac{\gamma}{2} + \chi_1\cos 2(\phi_1+\phi_2) - \sigma\right] \\ &\quad + \sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}\left(\eta_1\cos^2\frac{\gamma}{2} + \xi_1\sin^2\frac{\gamma}{2} - \chi_1 - \sigma\right) \\ &\quad + \frac{(\beta-\sigma)}{2}\sin\gamma\sin\theta_1\sin\theta_2\sin(\phi_1+\phi_2) + \sigma \end{aligned} \tag{12a}$$

$$\begin{aligned}\$_B(\theta_1,\phi_1,\theta_2,\phi_2) &= \cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\left[\xi_1\sin^2\frac{\gamma}{2} + \eta_1\cos^2\frac{\gamma}{2} - \chi_1\cos 2(\phi_1+\phi_2) - \sigma\right] \\ &\quad + \sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}\left(\xi_1\cos^2\frac{\gamma}{2} + \eta_1\sin^2\frac{\gamma}{2} + \chi_1 - \sigma\right) \\ &\quad + \frac{(\alpha-\sigma)}{2}\sin\gamma\sin\theta_1\sin\theta_2\sin(\phi_1+\phi_2) + \sigma, \end{aligned} \tag{12b}$$

where $\xi_1 = \alpha\cos^2\frac{\gamma}{2} + \beta\sin^2\frac{\gamma}{2}$, $\eta_1 = \alpha\sin^2\frac{\gamma}{2} + \beta\cos^2\frac{\gamma}{2}$ and $\chi_1 = \frac{(\alpha-\beta)}{2}\sin^2\gamma$. To draw a better comparison we take $\delta = \gamma = \frac{\pi}{2}$ then the payoffs given by equations (12) reduce to

$$\begin{aligned}\$_A(\theta_1,\phi_1,\theta_2,\phi_2) &= (\alpha-\sigma)\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\sin^2(\phi_1+\phi_2) \\ &\quad + (\beta-\sigma)\left[\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\sin(\phi_1+\phi_2) + \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right]^2 + \sigma \end{aligned} \tag{13a}$$

$$\begin{aligned}\$_B(\theta_1,\phi_1,\theta_2,\phi_2) &= (\alpha-\sigma)\left[\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\sin(\phi_1+\phi_2) + \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right]^2 \\ &\quad + (\beta-\sigma)\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\sin^2(\phi_1+\phi_2) + \sigma. \end{aligned} \tag{13b}$$

The payoffs given in equations (13) have already been found by Du *et al* [16] through the Eisert *et al* scheme [4].

(ii) $\phi_1 = \phi_2 = 0$ then, as shown by Eisert *et al* [4, 17], one gets classical payoffs with mixed strategies when one parameter set of strategies is used for the quantization of the game. For a better comparison putting $\gamma = \delta = \frac{\pi}{2}$ and $\phi_1 = \phi_2 = 0$ in equations (12*a*) and (12*b*) the same situation occurs and the payoffs reduce to

$$\begin{aligned}\$_A(\theta_1,\phi_1,\theta_2,\phi_2) &= \alpha\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2} + \beta\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2} \\ &\quad + \sigma\left(\cos^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2} + \sin^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\right) \end{aligned} \tag{14a}$$

$$\begin{aligned}\$_B(\theta_1,\phi_1,\theta_2,\phi_2) &= \beta\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2} + \alpha\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2} \\ &\quad + \sigma\left(\cos^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2} + \sin^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\right). \end{aligned} \tag{14b}$$

In this case the game behaves just like classical game where the players are playing mixed strategies with probabilities $\cos^2\frac{\theta_1}{2}$ and $\cos^2\frac{\theta_2}{2}$ respectively.

Case (c). When $\delta \neq \gamma$ and $\phi_1 = 0$, $\phi_2 = 0$, then payoffs given by the equations (9*a*), (9*b*) reduce to

$$\begin{aligned}\$_A(\theta_1, \phi_{1,}\theta_2, \phi_2) = {} & \cos^2\frac{\theta_1}{2}\left[\cos^2\frac{\theta_2}{2}(\alpha+\beta-2\sigma) - \alpha\sin^2\frac{(\gamma-\delta)}{2} - \beta\cos^2\frac{(\gamma-\delta)}{2} + \sigma\right] \\ & + \cos^2\frac{\theta_2}{2}\left[-\alpha\sin^2\frac{(\gamma-\delta)}{2} - \beta\cos^2\frac{(\gamma-\delta)}{2} + \sigma\right] \\ & + \alpha\sin^2\frac{(\gamma-\delta)}{2} + \beta\cos^2\frac{(\gamma-\delta)}{2} \end{aligned} \tag{15a}$$

$$\begin{aligned}\$_B(\theta_1, \phi_{1,}\theta_2, \phi_2) = {} & \cos^2\frac{\theta_1}{2}\left[\cos^2\frac{\theta_2}{2}(\alpha+\beta-2\sigma) - \beta\sin^2\frac{(\gamma-\delta)}{2} - \alpha\cos^2\frac{(\gamma-\delta)}{2} + \sigma\right] \\ & + \cos^2\frac{\theta_2}{2}\left[-\beta\sin^2\frac{(\gamma-\delta)}{2} - \alpha\cos^2\frac{(\gamma-\delta)}{2} + \sigma\right] \\ & + \beta\sin^2\frac{(\gamma-\delta)}{2} + \alpha\cos^2\frac{(\gamma-\delta)}{2}. \end{aligned} \tag{15b}$$

These payoffs are equivalent to that of Marinatto and Weber [5] when γ replaced with $\gamma - \delta$.

Case (d). When $\delta \neq 0$ and $\gamma = 0$ then from equations (9*a*), (9*b*) the payoffs of the players reduce to

$$\begin{aligned}\$_A(\theta_1, \phi_1, \phi_2, \theta_2) = {} & \cos^2\frac{\theta_1}{2}\left[\cos^2\frac{\theta_2}{2}(\alpha+\beta-2\sigma) - \alpha\sin^2\frac{\delta}{2} - \beta\cos^2\frac{\delta}{2} + \sigma\right] \\ & + \cos^2\frac{\theta_2}{2}\left(-\alpha\sin^2\frac{\delta}{2} - \beta\cos^2\frac{\delta}{2} + \sigma\right) + \alpha\sin^2\frac{\delta}{2} + \beta\cos^2\frac{\delta}{2} \\ & - \frac{(\alpha-\beta)}{2}\sin\delta\sin\theta_1\sin\theta_2\sin(\phi_1+\phi_2) \end{aligned} \tag{16a}$$

$$\begin{aligned}\$_B(\theta_1, \phi_1, \phi_2, \theta_2) = {} & \cos^2\frac{\theta_2}{2}\left[\cos^2\frac{\theta_1}{2}(\alpha+\beta-2\sigma) - \beta\sin^2\frac{\delta}{2} - \alpha\cos^2\frac{\delta}{2} + \sigma\right] \\ & + \cos^2\frac{\theta_1}{2}\left(-\beta\sin^2\frac{\delta}{2} - \alpha\cos^2\frac{\delta}{2} + \sigma\right) + \beta\sin^2\frac{\delta}{2} + \alpha\cos^2\frac{\delta}{2} \\ & + \frac{(\alpha-\beta)}{2}\sin\delta\sin\theta_1\sin\theta_2\sin(\phi_1+\phi_2). \end{aligned} \tag{16b}$$

This shows that the measurement plays a crucial role in quantum games as even if the initial state is unentangled, i.e., $\gamma = 0$, arbiter can still apply entangled basis for the measurement to obtain quantum mechanical results. The above payoffs are similar to that of Marinatto and Weber for the Battle of the Sexes games if δ is replaced by γ.

3. Conclusion

A generalized quantization scheme for non-zero sum games is proposed. The game of Battle of the Sexes has been used as an example to introduce this quantization scheme. However, our quantization scheme is also applicable to other games. This new scheme reduces to Eisert *et al*'s [4] scheme under the condition

$$\delta = \gamma, \quad \phi_1 + \phi_2 = \pi/2$$

and to Marinatto and Weber's [5] scheme when

$$\delta = 0, \qquad \phi_1 = 0, \qquad \phi_2 = 0.$$

In the above conditions γ is a measure of entanglement of the initial state. For $\gamma = 0$, classical results are obtained when $\delta = 0, \phi_1 = 0, \phi_2 = 0$. Furthermore, we have identified some interesting situations which are not apparent within the two existing quantization schemes. For example, with $\delta \neq 0$, non-classical results are obtained for an initially unentangled state. This shows that the measurement plays a crucial role in quantum games.

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